A New Global Finite Element Analysis of Microwave Circuits Including Lumped Elements

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***Abstract*—A new fullwave global analysis of complex inhomogeneous microwave structures including passive or active, linear or nonlinear lumped elements is presented. For a given structure, only one electromagnetic simulation of the distributed part, by a three-dimensional (3-D) finite element method using edge elements, is needed corresponding to the insertion of variable lumped elements placed at the same position. Simulation results of various test cases, containing either resistor, diode, or active component, are compared with those provided by a commercial circuit analysis software or with measurements results. Our validated electromagnetic simulator is then used for analyzing a planar balanced mixer operating in the millimetric range.**

I. Introduction

SINCE few years, thanks to the continuous improvement of computer calculation capacity, much attention has been focused on the investigation of complex microwave circuits, based on a numerical solution of Maxwell's equations solved for the whole circuit. By complex circuits we mean, those composed of distributed elements like various transmission lines and discontinuities and of lumped ones like resistors, capacitors, diodes or other active devices. A complete analysis of such circuits is generally performed by dividing the structure into several parts, that are studied independently from each other and recombined from a circuit point of view to give global results. This technique may be a simple one providing accurate results when reference planes separating different domains are easily defined and when electromagnetic coupling between different circuit parts can be neglected. But these conditions are not achieved in complex and highly integrated present microwave systems, operating at very high frequencies, such as millimeter-wave integrated circuits. An accurate fullwave electromagnetic (EM) modeling tool taking into account EM coupling between distributed parts and lumped components becomes necessary in these cases to get the whole circuits performances.

Few previous proposals for such global EM simulations are found in the literature. For instance, some authors have proposed to extend the general EM equations used in a numerical method, like the finite-difference time-domain (FDTD) method [1], [11] or the transmission line matrix method [2], to include the lumped elements by specifying the appropriate I-V characteristic of the elements. Some others have employed an equivalent impedance for modeling the lumped element when using the spectral domain analysis [4] or the finite element method (FEM) [3]. But the major drawbacks of these techniques are their dependence on the type of the studied lumped element or their applicability only on planar microwave circuits. Recent solutions consist of combining a rigorous EM simulator, using either integral equation method (IEM) [5] or FDTD method [6], [7], [12], with a circuit analysis software to gain advantage of both modeling tools. But to our knowledge, no proposal has been given yet for such a technique using FEM.

Our paper presents a new global EM analysis for characterizing complex microwave circuits containing various lumped components, based on the use of the FEM and of a circuit modeling software. The proposed technique has of course the advantages that presents a three-dimensional (3-D) finite element analysis using edge elements [8], namely: it allows a characterization of arbitrary shaped structures without having parasitic solutions. Furthermore, our technique is shown to be a flexible and fast practical one for simulation of microwave devices containing variable lumped elements, as it can directly make use of all circuit software models and also as it requires only one EM simulation for the distributed circuit part. It means in particular that nonlinear elements are easily taken into account as we use available nonlinear models and techniques like the Harmonic Balance. Our technique is firstly validated on test cases, like resistor or diode connected to a microstrip line through a gap or like a Gunn diode amplifier. It is then applied on the study of a more complex microwave device, a planar balanced mixer.

II. Three-Dimensional FE Formulation with Lumped Elements

Let us recall that the variational procedure of weighted residuals, using Green's theorem, applied on Maxwell curl equations [10] gives the following curl curl equation for the E-field formulation:

$$\frac{1}{j\omega\mu_0}\int_V \frac{1}{\mu_r}\mathrm{curl}\vec{E}.\mathrm{curl}\vec{E'} + j\omega\varepsilon_0\int_V \varepsilon_r\vec{E}.\vec{E'} = \int_{\delta V}\left(\vec{H}\times\vec{n}\right).\vec{E'} \qquad (1)$$

where V is a studied volume, δV is its boundary surface and $\vec{n}$ is the unit inward field vector normal to δV.

For discretizing (1), the volume V is meshed by tetrahedral elements. Edge elements have been shown to be adequate basis vectorial fields to approximate the electric field, since they provide important required EM properties [8]. Especially,

Manuscript received April 1, 1996.

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Publisher Item Identifier S 0018-9480(96)08571-7.

they only enforce the tangential continuity of the field across the tetrahedral element's boundaries, which is a necessary condition when dealing with dielectric interface. The electric field $\vec{E}$ can be then rewritten as:

$$\vec{E} = \sum_{e \in \{edges\}} e_e \vec{w}_e \tag{2}$$

where the set of $\vec{w}_e$, edge elements related to edge e, are the basis vectorial fields, defined for the volume V, and the set of e_e are the unknowns or the degrees of freedom (dof's) of the EM problem. In fact, an edge element can be thought of, as a probing function of the electric field along the edges of the tetrahedral elements, satisfying the condition

$$\int_{edge\, e} \vec{w}_e.\vec{dl} = \delta_{e,e'}. \tag{3}$$

This property means that the circulation of the edge element $\vec{w}_e$ equal 1 along the edge e and equal 0 along all others e'. Then the circulation of the electric field $\vec{E}$ along the edge e is given by

$$\begin{aligned}\int_{edge\, e} \vec{E}.\vec{dl} &= \int_{edge\, e} \sum_{e \in \{edges\}} e_e \vec{w}_e.\vec{dl}\\ &= \sum_{e \in \{edges\}} e_e \int_{edge\, e} \vec{w}_e.\vec{dl}\\ &= e_e.\end{aligned} \tag{4}$$

Consequently, the dof's are directly the circulations of the field along the edges.

Using these edge elements as basis functions in the Galerkin's method, (1) leads to the following matrix equation:

$$\begin{bmatrix} Y_{11} & \cdots & & Y_{1n} \\ \vdots & \ddots & & \vdots \\ \vdots & Y_{ee} & & \vdots \\ \vdots & & \ddots & \vdots \\ Y_{n1} & \cdots & & Y_{nn} \end{bmatrix} \cdot \begin{bmatrix} e_1 \\ \cdots \\ e_e \\ \cdots \\ e_n \end{bmatrix} = \begin{bmatrix} i_1 \\ \cdots \\ i_e \\ \cdots \\ i_n \end{bmatrix}$$

or

$$[Y] \cdot [e] = [i] \tag{5}$$

where $[e]$ is the vector of dof's (in volts), $[i]$ is the excitation current vector (in amperes) and $[Y]$ is an admittance matrix which relates the interaction between edges. This admittance matrix is assembled from the volumic integrals in the tetrahedral elements while the excitation current vector results from the boundary conditions of the EM problem, as expressed in both sides of (1).

According to circuit definitions, one edge e can be directly compared to a port whose own voltage and current characteristics are, respectively, e_e and i_e (Fig. 1). We can write

$$i_e = \sum_{k=1}^{M} Y_{ek} e_k, \quad \text{where } M \text{ is the number of dof's.} \tag{6}$$

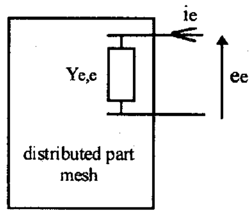

Fig. 1. Edge compared to an external usual port.

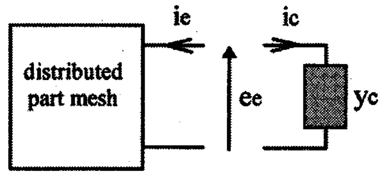

Fig. 2. Lumped element connected to an edge.

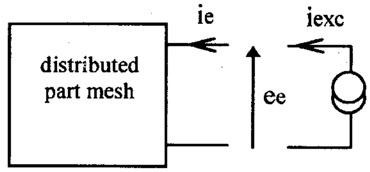

Fig. 3. Edge connected to a current source.

The matrix term Y_{ee} can be seen as the self admittance of the edge e, while a matrix term Y_{ek} gives the mutual admittance between the edge e and the edge k.

Then, for edges on excitation boundaries, the corresponding ports are connected to a given current source (Fig. 3). For the others, the inward port current is set to zero.

Now, if a lumped component is connected to this kind of port (Fig. 2), the Kirchhoff's laws can be written as

$$i_e + i_c = 0 \text{ where } i_c \text{ is the current in the lumped component.}$$

This law may be rewritten in two different ways

$$\sum_{k=1}^{M} Y_{ek} \cdot e_k + y_c \cdot e_e = 0 \tag{7a}$$

or

$$\begin{aligned}\sum_{k=1}^{M} Y_{ek} \cdot e_k &= -y_c \cdot e_e\\ &= -i_c.\end{aligned} \tag{7b}$$

The first form given in (7a) is used to introduce in the admittance matrix $[Y]$, an additional term that corresponds to the admittance y_c of the lumped element, as is done in the moment method matrix [5]. For instance, a resistor R, a capacitor C, or an inductor L connected to an edge e modifies the self admittance of the edge e, as

$$Y'_{e,e} = Y_{e,e} + \frac{1}{R},$$
$$Y'_{e,e} = Y_{e,e} + jC\omega$$

or

$$Y'_{e,e} = Y_{e,e} + \frac{1}{jL\omega} \tag{8}$$

respectively.

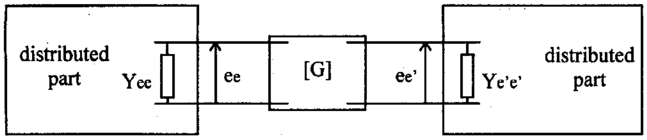

Fig. 4. Quadripole connected to the distributed circuit part.

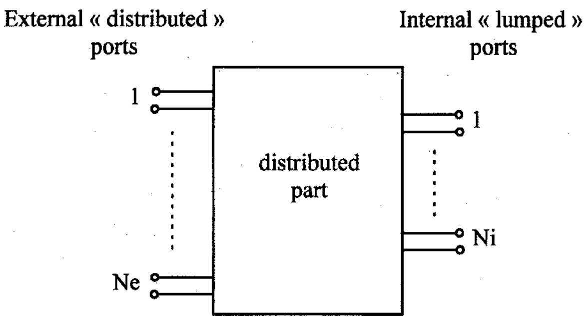

Fig. 5. Distributed circuit part characterized by its external and internal ports.

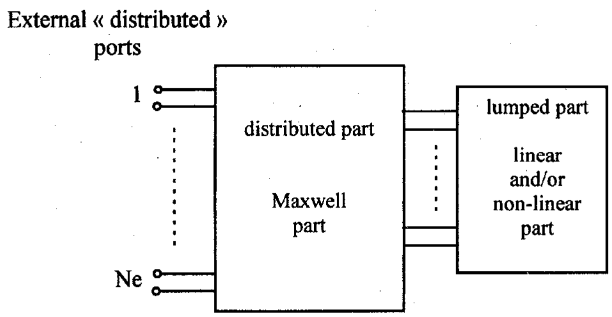

Fig. 6. Connection between the distributed part and the lumped part.

In the case of a quadripole having a known admittance matrix $[G]$, connected to the distributed part circuit through an edge e for its first port and an edge e' for its second port (Fig. 4), the admittance matrix of the circuit distributed part is modified as

$$\begin{aligned} Y'_{e,e} &= Y_{e,e} + G_{11} \\ Y'_{e,e'} &= Y_{e,e'} + G_{12} \\ Y'_{e',e} &= Y_{e',e} + G_{21} \\ Y'_{e',e'} &= Y_{e',e'} + G_{22} \end{aligned}$$

where

$$[G] = \begin{bmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{bmatrix}$$

is the admittance matrix of the lumped quadripole.

But the use of this method is restricted to simple lumped components, whose parameter y_c (or admittance matrix) can be obtained easily. Moreover a complete electromagnetic simulation has to be performed each time the value of the lumped component changes. Finally, a nonlinear lumped element seems to be very difficult to be inserted in the distributed part using this method.

We prefer to use the second formulation suggested in (7b). The inclusion of the lumped element is taken into account

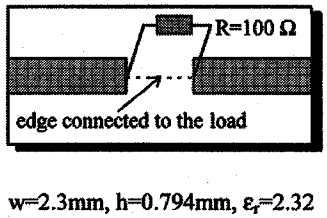

Fig. 7. Resistor connected to a microstrip line.

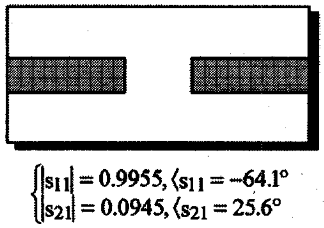

Fig. 8. Comparison with the gap.

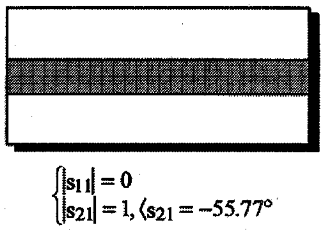

Fig. 9. Comparison with the microstrip line.

in the excitation current vector, as it consists of a boundary condition on an edge. In a first step, the edge corresponding to the location of the component will provide a new degree of freedom for characterizing the circuit. The edge is considered as a port, that we may call: an internal port because it is located in the heart of the structure. But it is treated like other usual external ports. It is either excited by a unit current source (Fig. 3), while the other ports are open-circuited, or open-circuited, while another port is excited. The conventional FEM applied on the distributed part of the circuit provides an impedance matrix that is converted to a scattering matrix (Fig. 5).

This matrix of reduced size (Ne + Ni), compared to the FEM one $[Y]$, summarizes or compresses [7] the Maxwell informations of the distributed part. Coupling effects, in particular around the lumped element location are rigorously expressed in this matrix. From a circuit point of view, this matrix characterizes the equivalent Thevenin/Norton circuit of the distributed part. In a second step, this matrix is connected to the lumped element using a circuit simulation software (Fig. 6). The final result is the usual scattering matrix of the complete device.

This technique presents several advantages. Indeed, its formulation is quite simple, direct and general: the insertion of any linear or nonlinear, passive or active components can be considered. Moreover, the lumped components can be as close as desired to circuit discontinuities, since electromagnetic coupling is taken into account. Finally, these lumped elements can

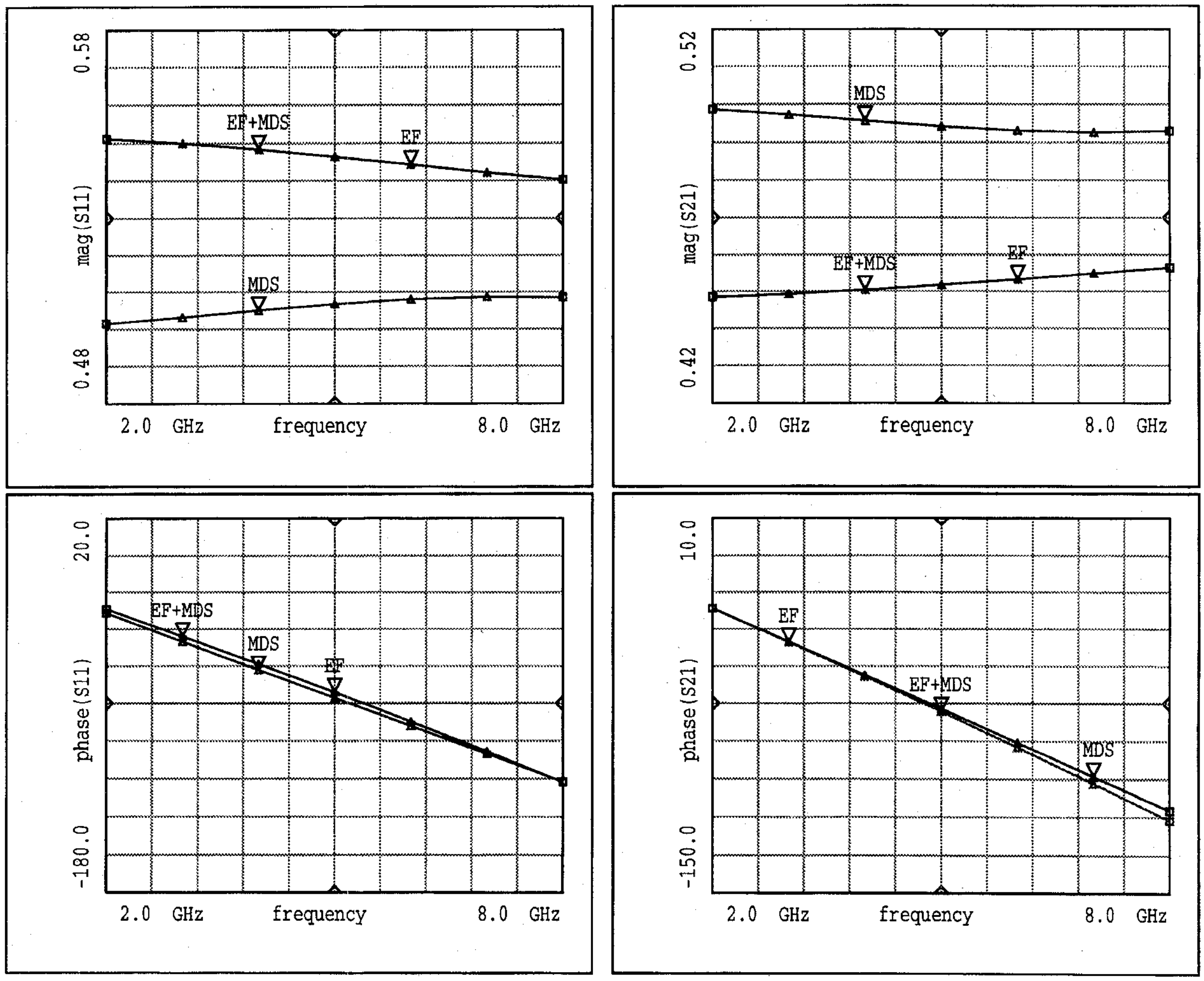

Fig. 10. S-parameters of the resistor connected to the microstrip line through a gap.

be changed without needing further electromagnetic simulation for the circuit distributed part.

III. Test Cases

Three simulations are compared in the following studies: the first one is entirely based on the use of only the commercial circuit analysis software Hewlett Packard (MDS), the second one is purely an electromagnetic analysis using directly the admittance matrix $[Y]$ of the finite element method, as explained above in (5) and (7a) (named EF), the third one is our technique that uses the Finite Element Method to simulate the distributed part and a circuit analysis tool to add the lumped component characteristics (named EF + MDS).

First, we simulate a 100-Ω resistor connected to a microstrip line through a gap (Fig. 7). The results give the variations of the magnitude and the phase of the S-parameters as a function of frequency (Fig. 10). The comparison between our simulation (EF + MDS) and the second one (EF) shows the equivalence of both approaches and their difference from the equivalent circuit model approach (MDS). It proves the importance of a global electromagnetic simulation even for relatively non complicated circuits.

The variations of the S-parameters are studied when the resistor value changes from 1 to 100 kΩ at a frequency of 4 GHz (Fig. 11). Note that, contrary to the MDS and EF + MDS simulations, the second method requires a new electromagnetic simulation for each new resistance value. As expected, both other simulations are equivalent when the resistor value is very large: the S-parameters converge toward those characterizing the gap (Fig. 8). But if the resistor value is very small, results of the two simulations differ: the circuit analysis results (MDS) are closer to those characterizing the simple microstrip line without gap (Fig. 9), while the electromagnetic simulation (EF + MDS) shows more reflections because the discontinuity due to the lumped element is taken into account. The same remark was reported in [4] when using the moment method for a test case of a coplanar waveguide loaded by a resistor.

To validate our technique for the case of the insertion of nonlinear element, the test case shown in (Fig. 12) is analyzed at a low frequency of 4 GHz, in order to be able to neglect the effect of the coupling between localized and distributed circuit elements. It consists of a diode connected to a microstrip line through an air-bridge. Results presented for the diode voltage and current responses are shown in (Fig. 13). There is a good agreement between the results calculated by our combined

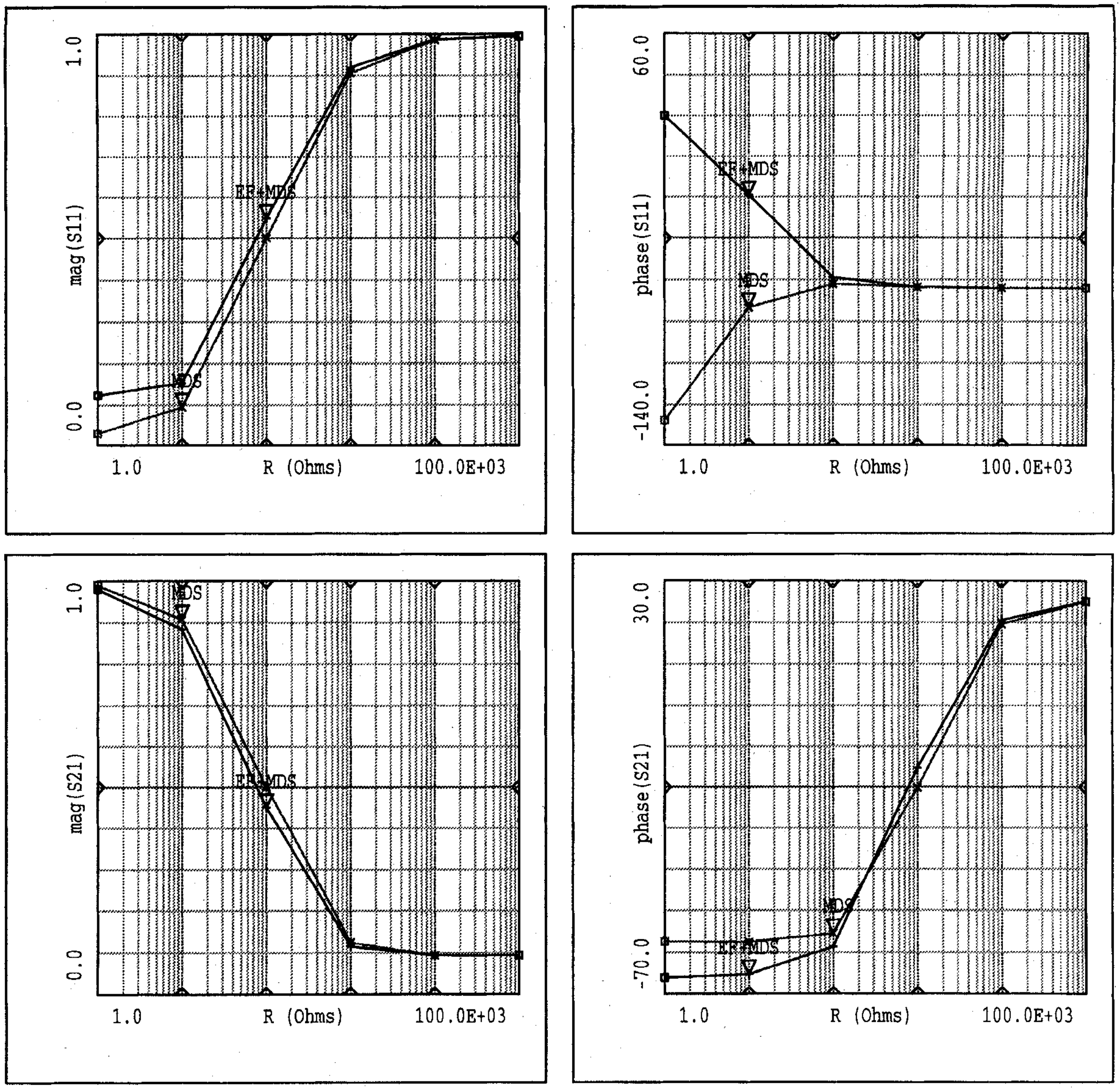

Fig. 11. Variations of the S-parameters with the resistor value for the example of (Fig. 7).

simulator and those calculated using the nonlinear circuit analysis tool (MDS). These results prove the adaptability of our technique to characterize circuits containing linear or nonlinear lumped elements.

Finally, our technique is used to simulate an active microwave structure: a linear Gunn diode amplifier (Fig. 14). Our results (Fig. 15) are in a very good agreement with the measurements published in [3]. A deviation of less than 1.2% is reported between the two results.

Through the above presentation of some test cases, our simulator is shown to be very well adapted for the modeling of complex structures containing linear or nonlinear, passive or active devices.

IV. Application to the Analysis of a Planar Balanced Mixer

We choose to study a planar balanced mixer operating in the millimetric range, that can be hardly simulated by available

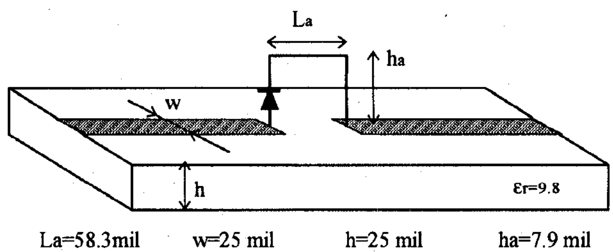

Fig. 12. An air-bridge connecting a diode to a microstrip line.

commercial electromagnetic or circuit simulators, since there is no possible separation between the lumped nonlinear element and the distributed circuit. Moreover, the use of a global EM simulator such as ours is well justified for the analysis of this mixer, since the effect of the insertion of the lumped elements becomes important in the millimetric range.

The studied mixer is a 26.5–40-GHz band balanced fin line mixer [9], fabricated on a 0.254-mm-thick Duroïd 5880

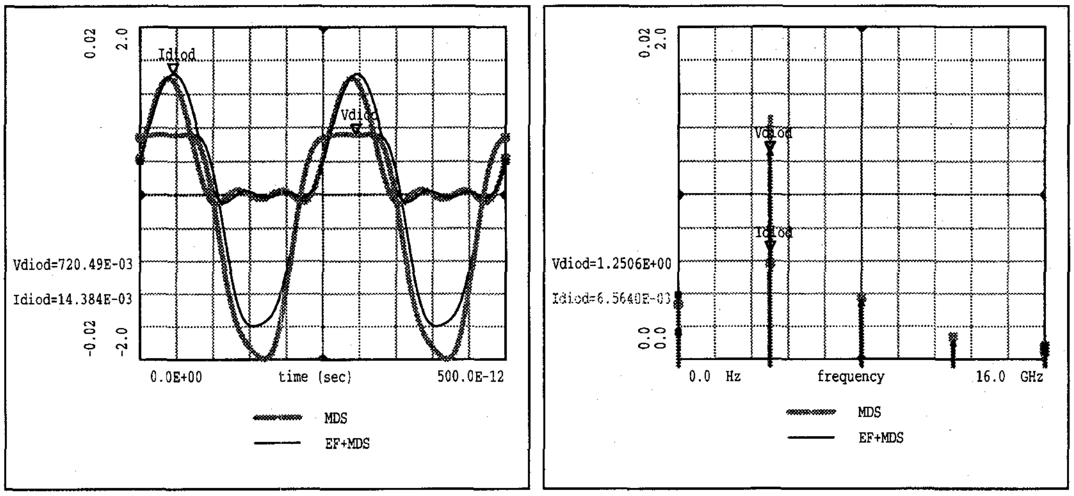

Fig. 13. Temporal and spectral responses of diode voltage and current for the structure of Fig. 12.

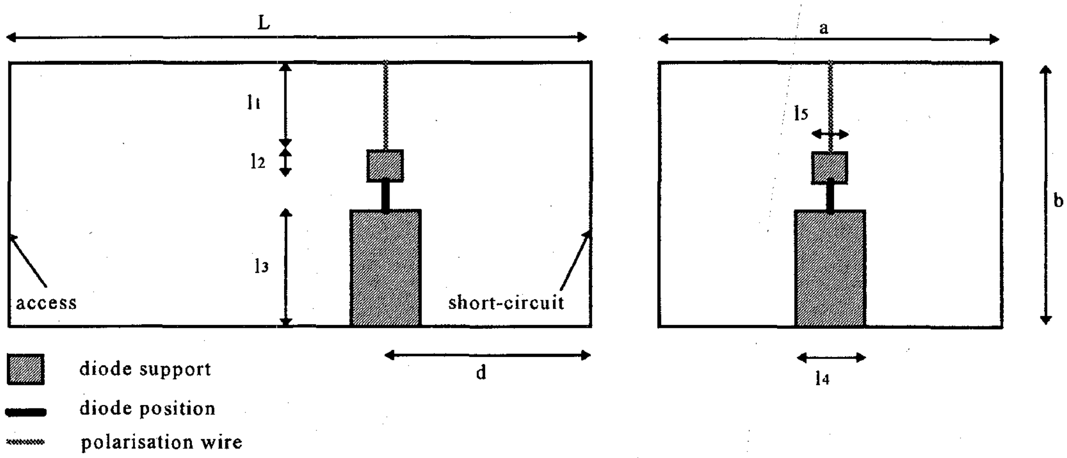

Fig. 14. Gunn diode amplifier.

substrate. It uses a planar magic tee composed of a slotline–microstrip line junction. Two Schottky diodes are embedded in the heart of the junction (Fig. 16). In our simulation, the microstrip line provides a LO power of 11.5 dBm at a frequency of 24.75 GHz. The slotline is used to feed the RF signal at a frequency in the 27.25–29.25 GHz band, for a power level of -10 dBm. The microstrip line is also used to get the output signal, through a via hole.

The frequency spectrum of the output signal power in dBm, given by our FEM–MDS simulations, is shown in (Fig. 17) for an RF frequency of 28.5 GHz. Since the mixer is a balanced one, very low levels of the harmonics at 7.5 GHz ($2f_{\mathrm{RF}} - 2f_{\mathrm{LO}}$) and at 49.5 GHz ($2f_{\mathrm{LO}}$) are observed (respectively -75 dBm and -68 dBm). Moreover, the image frequency at 21 GHz ($2f_{\mathrm{LO}} - f_{\mathrm{RF}}$) is also at a very low level (-70 dBm) at the output port, compared to the IF level (-10.04 dBm at 3.75 GHz). The simulated spectrum is in a good agreement with the theoretical predictions.

These comparisons confirm the adaptability of our EM simulator for the treatment of difficult problems, like that of planar balanced mixers operating in the millimetric range.

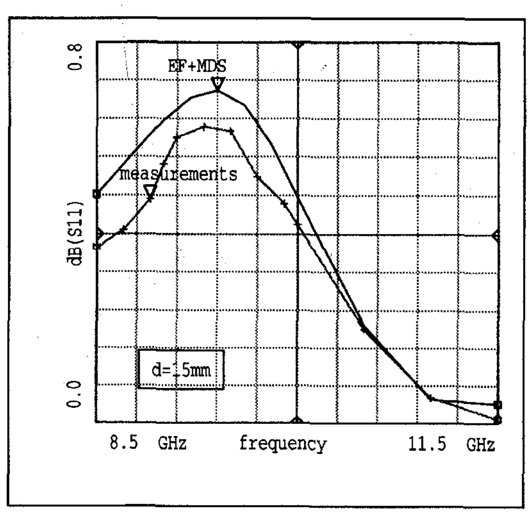

Fig. 15. Comparisons between our results and the measurements for the structure of (Fig. 14). (a = 22.86 mm, b = 10.16 mm, L = 35 mm, l_1 = 6.16 mm, l_2 = 0.25 mm, l_3 = 3.2 mm, l_4 = 4 mm, l_5 = 1.3 mm).

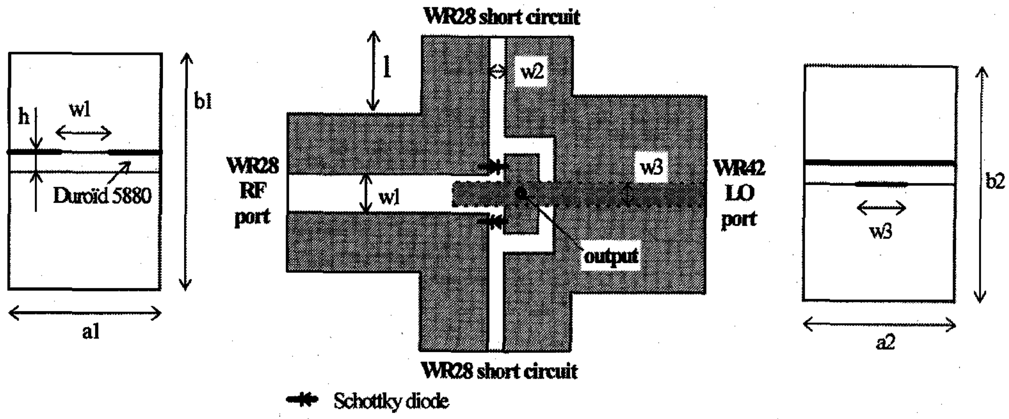

Fig. 16. 26.5–40-GHz band balanced finline mixer. ($a1 = 3.556$ mm, $b1 = 7.112$ mm, $a2 = 4.318$ mm, $b2 = 10.668$ mm, $h = 0.254$ mm, $w1 = 0.603$ mm, $w2 = 0.05$ mm, $w3 = 0.572$ mm, $l = 1.172$ mm).

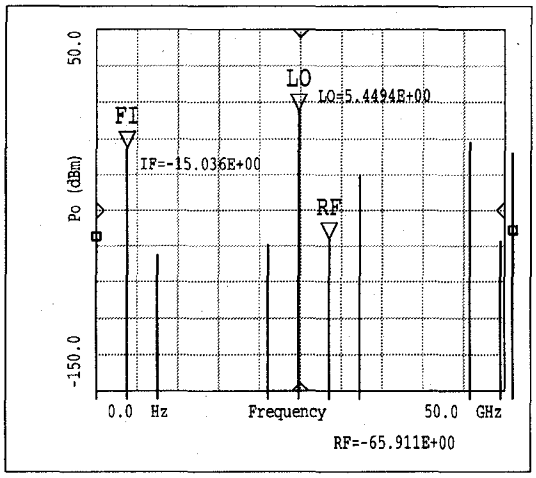

Fig. 17. Mixer output power for the 26.5–40 GHz band balanced mixer.

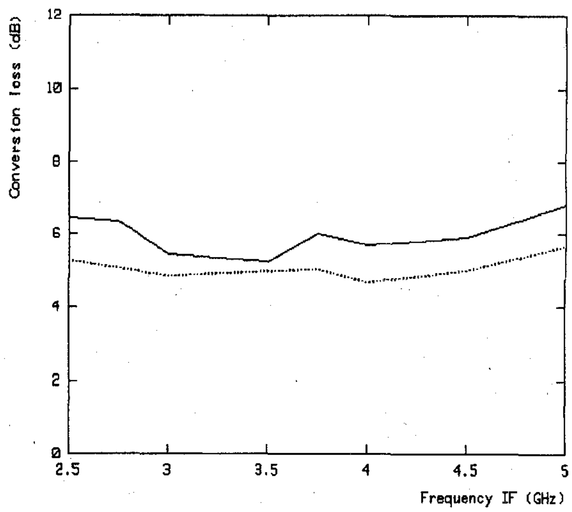

Fig. 18. Conversion loss in the 2.5–5 GHz IF band, measurements (continuous), simulation (dashed).

As shown in (Fig. 18), the conversion loss given by our simulations for an IF frequency in the 2.5–4.5-GHz range agrees with the measured conversion loss of the whole mixer.

V. Conclusion

A new efficient method is presented to simulate the insertion of any lumped passive or active, linear or nonlinear components into a distributed microwave structure. Based on the use of a 3-D FEM using edge elements, our technique can be used to characterize any structure geometry, taking into account EM coupling between lumped and distributed parts. The distributed part is rigorously and efficiently characterized once before the insertion of lumped linear or nonlinear circuit elements in a circuit simulator, using appropriate linear or nonlinear circuit analysis software. Our proposed technique avoids the repetition of heavy numerical calculations, corresponding to the EM characterization of the circuit distributed part, when the lumped elements have to be changed. This is particularly useful when optimization of variable lumped elements is needed. The validity and precision of our formulation is shown through the presentation of some test cases including the insertion of linear/nonlinear localized elements.

Acknowledgment

The authors would like to thank Prof. Guillon and his group of IRCOM Laboratory, Limoges, France, for providing the measurement results of the Gunn diode amplifier.

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